

7

THE X-WING FAMILY

The X-wing, and its sister solving methods (swordfish, jellyfish, and squirmbag), are found in some of the hardest sudoku puzzles.

X-WING

Here is a puzzle that uses an X-wing:

Example 13

9			3					7
5			9			2		
				2	4	3		
					8		6	
4	6			9			1	3
	9		4					
		3	5	7				
		1			9			2
7					2			5

Using basic strategies, we get stuck at this point:

Example 13-1

9	¹²	²⁴⁶	3	¹⁵⁸	¹⁵⁶	¹⁶	⁴⁸	7
5	3	⁴⁶⁷	9	¹⁸	¹⁶⁷	2	⁴⁸	¹⁶
8	¹⁷	⁶⁷	¹⁶⁷	2	4	3	5	9
3	²⁷	²⁵⁷	¹⁷	¹⁵	8	9	6	4
4	6	8	2	9	⁵⁷	⁵⁷	1	3
1	9	⁵⁷	4	6	3	⁵⁷	2	8
2	4	3	5	7	¹⁶	8	9	¹⁶
6	5	1	8	3	9	4	7	2
7	8	9	¹⁶	4	2	¹⁶	3	5

In columns 2 and 4, the only cells with 7 candidates are 32, 42, 34, and 44. Because these four cells lie in only two different rows (3 and 4), we know that 32 and 44 will contain 7's, or 34 and 42 will contain 7's. In either case, the 7's in rows 3 and 4 will appear in columns 2 and 4. In other words, if cell 33 contained a 7, then neither cell 32 or 34 could contain a 7, so then cells 42 and 44 would both have to be 7's, but that's impossible, since they're in the same row. This grouping is known as an X-wing, and gets its name from the shape formed by connecting cells 34 to 42 and cells 32 to 44, which forms an X. Because of the X-wing, we can eliminate all the 7 candidates wherever else they appear in rows 3 and 4. That means that cell 33 has to be a 6.

Example 13-2

9	12	24	3	158	156	16	48	7
5	3	47	9	18	167	2	48	16
8	17	6	17	2	4	3	5	9
3	27	25	17	15	8	9	6	4
4	6	8	2	9	57	57	1	3
1	9	57	4	6	3	57	2	8
2	4	3	5	7	16	8	9	16
6	5	1	8	3	9	4	7	2
7	8	9	16	4	2	16	3	5

After the 6 is in place at cell 33, another 6 can be placed in cell 94 with elimination (a 6 can't go anywhere else in column 4), and it's smooth sailing from there using basic strategies.

Example 13 Answer

9	2	4	3	1	5	6	8	7
5	3	7	9	8	6	2	4	1
8	1	6	7	2	4	3	5	9
3	7	2	1	5	8	9	6	4
4	6	8	2	9	7	5	1	3
1	9	5	4	6	3	7	2	8
2	4	3	5	7	1	8	9	6
6	5	1	8	3	9	4	7	2
7	8	9	6	4	2	1	3	5

SWORDFISH

Swordfish is similar to X-wing, except it involves three rows (or columns) at once, instead of two. The origins of the names of this and higher-level X-wings are a mystery. Here's a puzzle using swordfish:

Example 14

	3							
			9		3	2		1
6	7			2			3	
9			8		7			
		7				5		
			3		6			7
	8			9			1	6
1		2	5		8			
							2	

We get this far with the basics:

Example 14-1

2	3	14589	1467	14567	145	4679	45678	489
458	45	458	9	45678	3	2	45678	1
6	7	14589	14	2	145	49	3	489
9	1456	13456	8	145	7	1346	46	2
348	146	7	2	14	9	5	468	348
458	2	1458	3	145	6	149	489	7
3457	8	345	47	9	2	347	1	6
1	469	2	5	3467	8	3479	479	349
347	469	3469	1467	13467	14	8	2	5

Now we need to use interaction. The 1 in column 2 will be in either cell 42 or 52, so we can remove the 1 candidates from the other cells in box 4. Similarly, the 9 in column 2 will be in cell 82 or 92, so the 9 can be removed from the candidate list in cell 93. The 9 candidate in cell 82 can be removed because a 9 has to be in row 8 of box 9. The 1, 4, and 5 form a triplet in cells 45, 55, and 65, so 1, 4, and 5 can be removed from the rest of that column.

Example 14-2

2	3	14589	1467	678	145	4679	45678	489
458	45	458	9	678	3	2	45678	1
6	7	14589	14	2	145	49	3	489
9	1456	3456	8	145	7	1346	46	2
348	146	7	2	14	9	5	468	348
458	2	458	3	145	6	149	489	7
3457	8	345	47	9	2	347	1	6
1	46	2	5	367	8	3479	479	349
347	469	346	1467	367	14	8	2	5

Now column 2 has just one 9 in it, so make cell 92 a 9. Cells 21, 22, and 23 form a triplet with 4, 5, and 8. All the 4's, 5's, and 8's can be removed from cells 13, 33, 25, and 28.

Example 14-3

2	3	19	1467	678	145	4679	45678	489
458	45	458	9	67	3	2	67	1
6	7	19	14	2	145	49	3	489
9	1456	3456	8	145	7	1346	46	2
348	146	7	2	14	9	5	468	348
458	2	458	3	145	6	149	489	7
3457	8	345	47	9	2	347	1	6
1	46	2	5	367	8	3479	479	349
347	9	346	1467	367	14	8	2	5

Using previously learned methods, we get to the grid in the next column (in row 1 a triplet allowed us to eliminate candidates 1, 4, and 9 in cells 14 and 17).

Example 14-4

2	3	19	67	8	14	67	5	49
458	45	458	9	67	3	2	67	1
6	7	19	14	2	5	49	3	8
9	1456	3456	8	145	7	1346	46	2
348	146	7	2	14	9	5	468	34
458	2	458	3	145	6	149	489	7
3457	8	345	47	9	2	347	1	6
1	46	2	5	367	8	3479	479	349
347	9	346	1467	367	14	8	2	5

Now we have a swordfish. In rows 2, 5, and 8, the only cells with 6's as candidates are 25, 28, 52, 58, 82, and 85. Because these cells lie in exactly three different columns (2, 5, and 8), we can eliminate the 6 candidates wherever else they appear in columns 2, 5, and 8.

Example 14 Swordfish

	○			○			○	
	○			○			○	
	○			○			○	

In the diagram above, the circles show where the 6's can go in rows 2, 5, and 8. There are two possible arrangements. One is with 6's in cells 25, 58, and 82. The other is with 6's in cells 28, 52, and 85. But in either case, a 6 will be in one of cells 52 and 82, one of cells 25 and 85, and one of cells 28 and 58. Therefore, the 6's are accounted for in columns 2, 5, and 8, and can be removed from the candidates in

those columns outside of rows 2, 5, and 8.

It is important to note that a swordfish doesn't have to have just two possible places for the 6 in each row. If cell 22 also had a 6 candidate, or cell 55, or cell 88 (or any combination of these), it would still be a swordfish. We'd know that the 6's in columns 2, 5, and 8 had to go somewhere in rows 2, 5, and 8, and could be eliminated from the other cells in those columns.

Removing the 6's, we get this:

Example 14-5

2	3	19	1467	8	14	4679	5	489
458	45	458	9	67	3	2	67	1
6	7	19	14	2	5	49	3	8
9	145	3456	8	145	7	1346	4	2
348	146	7	2	14	9	5	468	34
458	2	458	3	145	6	149	489	7
3457	8	345	47	9	2	347	1	6
1	46	2	5	367	8	3479	479	349
347	9	346	1467	37	14	8	2	5

By removing the 6 from cell 48 with the swordfish, we now know it has to be a 4, and the rest can be solved using basic strategies.

Example 14 Answer

2	3	9	6	8	1	7	5	4
8	5	4	9	7	3	2	6	1
6	7	1	4	2	5	9	3	8
9	1	3	8	5	7	6	4	2
4	6	7	2	1	9	5	8	3
5	2	8	3	4	6	1	9	7
3	8	5	7	9	2	4	1	6
1	4	2	5	6	8	3	7	9
7	9	6	1	3	4	8	2	5

JELLYFISH

Just as we went from pairs to triplets to quadruplets in the subsets section, here we move from X-wing to swordfish to jellyfish:

Example 15

	2			1	8			4
	5				6		3	2
7				5		8		9
			4		7			
6		2		9				5
2	9		3				4	
3			6	8			9	

We get stuck after using basic strategies at this point:

Example 15-1

9	2	3	7	1	8	6	5	4
148	1467	14678	25	3	25	9	18	178
18	5	178	9	4	6	17	3	2
7	134	14	12	5	123	8	6	9
5	8	9	4	6	7	13	2	13
6	13	2	8	9	13	4	7	5
2	9	168	3	7	15	15	4	168
148	1467	14678	15	2	9	1357	18	13678
3	17	5	6	8	4	2	9	17

Using interaction, we can eliminate some candidates. The 8's in cells 21 and 23 can be removed since we know that the 8 in row 3 will be in cell 31 or 33. Cells 76 and 77 form a pair and allow us to eliminate 1's in cells 73 and 79.

Example 15-2

9	2	3	7	1	8	6	5	4
¹⁴	¹⁴⁶⁷	¹⁴⁶⁷	²⁵	3	²⁵	9	¹⁸	¹⁷⁸
¹⁸	5	¹⁷⁸	9	4	6	¹⁷	3	2
7	¹³⁴	¹⁴	¹²	5	¹²³	8	6	9
5	8	9	4	6	7	¹³	2	¹³
6	¹³	2	8	9	¹³	4	7	5
2	9	⁶⁸	3	7	¹⁵	¹⁵	4	⁶⁸
¹⁴⁸	¹⁴⁶⁷	¹⁴⁶⁷⁸	¹⁵	2	9	¹³⁵⁷	¹⁸	¹³⁶⁷⁸
3	¹⁷	5	6	8	4	2	9	¹⁷

At this point we must use jellyfish to continue. In rows 5, 6, 7, and 9, the only cells with 1 candidates are 57, 59, 62, 66, 76, 77, 92, and 99. Because these eight cells lie in exactly four different columns (2, 6, 7, and 9), we can eliminate candidate 1's wherever else they appear in columns 2, 6, 7, and 9. This removes 1's from the cells 22, 42, 82, 46, 37, 87, 29, and 89.

Example 15 Jellyfish

						○		○
	○				○			
					○	○		
	○							○

This is just like the swordfish, but involving four rows instead of three. Note again that it doesn't have to be exactly two occurrences of each number in the rows for this to work. As long as they all fall in four columns, it doesn't matter if the rows have 2, 3, or 4 possible places for 1's to go.

Example 15-3

9	2	3	7	1	8	6	5	4
¹⁴	⁴⁶⁷	¹⁴⁶⁷	²⁵	3	²⁵	9	¹⁸	⁷⁸
¹⁸	5	¹⁷⁸	9	4	6	⁷	3	2
7	³⁴	¹⁴	¹²	5	²³	8	6	9
5	8	9	4	6	7	¹³	2	¹³
6	¹³	2	8	9	¹³	4	7	5
2	9	⁶⁸	3	7	¹⁵	¹⁵	4	⁶⁸
¹⁴⁸	⁴⁶⁷	¹⁴⁶⁷⁸	¹⁵	2	9	³⁵⁷	¹⁸	³⁶⁷⁸
3	¹⁷	5	6	8	4	2	9	¹⁷

With the 1 removed from cell 37, that becomes a 7. And putting a 7 in cell 37 makes cell 29 an 8, since the jellyfish also removed the 1 candidate from cell 29. Then we're home free with basic strategies.

Example 15 Answer

9	2	3	7	1	8	6	5	4
4	6	7	2	3	5	9	1	8
8	5	1	9	4	6	7	3	2
7	3	4	1	5	2	8	6	9
5	8	9	4	6	7	1	2	3
6	1	2	8	9	3	4	7	5
2	9	8	3	7	1	5	4	6
1	4	6	5	2	9	3	8	7
3	7	5	6	8	4	2	9	1

SQUIRMBAG

On to the squirmbag, which is just like a jellyfish except with one more level.

Example 16

1					8		4	
		4		3				9
	2		4		6		8	
		5	9	7		2		
		2		4	1	5		
	8		1		9		5	
5				8		7		
	4		3					1

You won't likely ever find a puzzle that requires you to use a squirmbag. They're extremely rare (even the jellyfish is quite rare). In this puzzle, it's not needed for solving, but we'll use it anyway, just to demonstrate the concept. Suppose you get to this point and you don't see any more obvious moves:

Example 16-1

1	3579	379	257	259	8	6	4	257
8	567	4	257	3	257	1	27	9
79	2	79	4	1	6	3	8	57
4	16	5	9	7	3	2	16	8
367	1367	8	256	256	25	9	1367	3467
3679	3679	2	8	4	1	5	367	367
2367	8	367	1	26	9	4	5	236
5	369	1	26	8	24	7	2369	236
2679	4	679	3	256	257	8	269	1

In columns 1, 3, 4, 5, and 9, the cells with 6's among the candidates are 51, 61, 71, 91, 73, 93, 54, 84, 55, 75, 95, 59, 69, 79, and 89. Because these cells lie in exactly five different rows (5, 6,

7, 8, and 9), we can eliminate the 6's wherever else they appear in rows 5, 6, 7, 8, and 9.

Here's a diagram of the cells with the 6's that form the squirmbag:

Example 16 Squirmbag

O			O	O				O
O								O
O		O		O				O
			O					O
O		O		O				

The cells that have their 6's eliminated are 52, 62, 82, 58, 68, 88, and 98. That leaves just one 6 in column 8, at cell 48. That 6 forces cell 42 to be a 1, and then column 2 has just one 6, at cell 22. From there the puzzle can be solved using scanning, one-choice, and elimination.

Example 16 Answer

1	5	3	7	9	8	6	4	2
8	6	4	5	3	2	1	7	9
7	2	9	4	1	6	3	8	5
4	1	5	9	7	3	2	6	8
3	7	8	6	2	5	9	1	4
6	9	2	8	4	1	5	3	7
2	8	7	1	6	9	4	5	3
5	3	1	2	8	4	7	9	6
9	4	6	3	5	7	8	2	1